SO(10) unified models and soft leptogenesis

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ABSTRACT: Motivated by the fact that, in some realistic models combining SO(10) GUTs and flavour symmetries, it is not possible to achieve the required baryon asymmetry through the CP asymmetry generated in the decay of right-handed neutrinos, we take a fresh look on how deep this connection is in SO(10). The common characteristics of these models are that they use the see-saw with right-handed neutrinos, predict a normal hierarchy of masses for the neutrinos observed in oscillating experiments and in the basis where the right-handed Majorana mass is diagonal, the charged lepton mixings are tiny. In addition these models link the up-quark Yukawa matrix to the neutrino Yukawa matrix Y^ν with the special feature of $Y^\nu_{11} \to 0$. Using this condition, we find that the required baryon asymmetry of the Universe can be explained by the soft leptogenesis using the soft B parameter of the second lightest right-handed neutrino whose mass turns out to be around 10^8 GeV. It is pointed out that a natural way to do so is to use no-scale supergravity where the value of $B \sim 1$ GeV is set through gauge-loop corrections.

KEYWORDS: Quark Masses and SM Parameters, GUT, Baryogenesis, Neutrino Physics.

Contents

1. Introduction

Some of the unanswered questions left by the Standard Model (SM) acquire a new light in Grand Unified Theories (GUTs). Among them the understanding of the structure of masses has an important role. Theories based on SO(10) seem to have a special place due to the possible structure of masses that can be achieved. First of all the spinorial **16** representation can accommodate all the known fermions including a right handed neutrino component, N, which makes it possible to embed in a natural way a see-saw mechanism for the explanation of the tiny masses of low energy neutrinos. On the other hand baryogenesis through leptogenesis [1] is a simple mechanism to explain the observed baryon asymmetry in the universe. Here a lepton asymmetry is dynamically generated and then converted

into a baryon asymmetry due to $B+L$ violating sphaleron interactions. The lightest of the right-handed neutrinos is produced by thermal scattering after inflation. It subsequently decays out-of-equilibrium to a lepton and a Higgs doublet producing a CP and lepton number violating asymmetry. The connection of leptogenesis to SO(10) GUTs is then natural, both by the inclusion of right handed neutrinos in the **16**s and by the possible variation of their masses which can be some orders of magnitude below $M_{\rm GUT} \sim 10^{16}$ GeV.

However in realistic models where the expansion parameter describing the Yukawa couplings of neutrinos, ϵ_ν is of the order of the expansion parameter describing the Yukawa couplings of up-type quarks, ϵ_u [2–4], the value needed for the mass of the lightest right-handed neutrino, $M_{N_1} \sim 10^{(7-8)}$ GeV, lies below the bounds for a successful thermal leptogenesis; $M_{N_1} \gtrsim 10^9$GeV [5]. This prompts us to address two questions as follows:

(i) How general is this statement in the context of Grand Unified models where one naturally gets

$$\epsilon_\nu \sim \epsilon_u \sim \sqrt{\frac{m_u}{m_c}} \ ? \tag{1.1}$$

(ii) If that is the case for many classes of such models, what are the possible scenarios for leptogenesis that we may require to consider in order to preserve such a feature of Grand Unified Models?

In the present work we show how easy it is to generate the relation eq. 1.1 and at the same time obtain $M_N \sim 10^{(7-8)}$ GeV for lighter right-handed neutrinos. This will be shown explicitly in the limiting case of the vanishing neutrino mixing angle associated with reactor experiments $s_{13} \to 0$, and of the dominant contribution of two light right-handed neutrinos. The contribution from non-zero s_{13} and the heaviest right-handed neutrino can be taken as perturbations. This enables us to determine, from the current experimental values, the possible form of the Dirac couplings of left-handed neutrinos in the basis where the mass of the right-handed neutrinos is diagonal, but without assuming any particular hierarchy among them.

We then embed these results in a SO(10) context, without fully specifying a model but rather making choices that are compatible with GUTs, which can be used as a starting point in the construction of a complete SO(10) model. In fact our results are compatible with the models of [2] (RV), [3] (CM), [4] (BKOT). Some results of the analysis are valid for the model [6] (DHR) however in this case all the masses of right handed neutrinos become relevant and hence the parameters relevant for leptogenesis behave differently. In all these models, one considers symmetric matrices at the scale where SO(10) has not been broken and then explains the generation of fermion masses with a minimal content of Higgs bosons, such as two **10** representations, which are the Higgs bosons in the u and d sectors, a $\overline{\mathbf{126}}$ representation to generate masses for right-handed neutrinos and a non-renormalizable operator **45** to distinguish some features of the fermion masses. These are generic features that may be used to construct more specific models, and in fact have been used extensively ([2]–[12]).

The neutrino Yukawa texture and the right-handed neutrino masses determined in many works do not allow for the standard leptogenesis [1] to produce the required baryon asymmetry of the Universe. Other mechanisms of inducing the appropriate baryon asymmetry in extensions of the MSSM may be implemented (e.g. [13] and see [14] for a review) or also mechanisms just using extra right-handed neutrinos (e.g. [44]).

However, we observe that the soft leptogenesis of [15], allowing a resonance condition with a small B term, works quite well within our scenario through the decay of the second lightest right-handed sneutrino whose mass is around 10^8 GeV. We will see that the required small B term ($B \sim 1\,\mathrm{GeV}$) arises from the gauge one-loop correction involving a heavy GUT gaugino once its tree-level value vanishes as it can be arranged easily in no-scale supergravity.

2. Possible forms of m_{LR}^ν and M_R in models with an underlying Grand Unified theory

We will identify elements of the right-handed neutrino mass matrix M_R and the Dirac neutrino mass matrix, $m_{LR}^\nu = Y^\nu v \sin\beta/\sqrt{2}$, which are related to the low energy observables (neutrino masses and mixing angles) by

$$m^\nu = U^\dagger \begin{bmatrix} m_{\nu_1} & & \\ & m_{\nu_2} & \\ & & m_{\nu_3} \end{bmatrix} U^* = -m_{LR}^\nu M_R^{-1} (m_{LR}^\nu)^T. \tag{2.1}$$

Here U is the neutrino mixing matrix and m_{ν_i} are the neutrino mass eigenvalues. This expression is valid in the basis where charged leptons are diagonal, if their matrix is not diagonal then we get $U = U^\nu U^{e*}$. Following the standard parameterization, let us write

$$U = U_{23} P_\delta^* U_{13} P_\delta U_{12} P_m \tag{2.2}$$

where U_{ij} is the rotation matrix in the (i,j) plane, $P_\delta = \mathrm{diag}[e^{-i\delta}, 1, e^{i\delta}]$ and $P_m = \mathrm{diag}[e^{i\rho}, e^{i\sigma}, 1]$ are the Dirac and Majorana phase matrices, respectively. We will use the notation $(m_{LR}^\nu)_{ij} = m_{ij}$ and $(Y^\nu)_{ij} = y_{ij}$, that is, $m_{ij} = y_{ij} v \sin\beta/\sqrt{2}$.

For numerical values, we use the latest values of the fits for neutrino oscillation observables [16]:

$$\Delta m_{21}^2 \in [7.3, 9.3] \times 10^{-5} \mathrm{eV}^2, \quad t_{12}^2 \in [0.28, 0.60]$$
$$\Delta m_{32}^2 \in [1.6, 3.6] \times 10^{-3} \mathrm{eV}^2, \quad t_{23}^2 \in [0.25, 2.1]$$
$$s_{13}^2 \leq 0.041. \tag{2.3}$$

Following the GUT relations consistent with the hierarchical pattern of all the fermion masses, we will work with the normal hierarchy;

$$m_{\nu_3} \gg m_{\nu_2} \gg m_{\nu_1}. \tag{2.4}$$

Then, we make the following definitions to help us trace the hierarchy of low-energy neutrino observables:

$$\mathbf{r} \equiv \sqrt{\frac{\Delta m_{21}^2}{\Delta m_{32}^2}} = \frac{m_{\nu_2}}{m_{\nu_3}}, \quad \mathbf{t} \equiv \frac{m_{\nu_1}}{m_{\nu_3}}. \qquad (2.5)$$

2.1 GUT considerations

Let us recall that the Yukawa sector of SO(10) at the renormalizable level comes from the following allowed couplings in the matter Lagrangian [17];

$$\mathcal{L}_M = y_{ij}^{10}(\mathbf{16})_i(\mathbf{16})_j(\mathbf{10}) + y_{ij}^{120}(\mathbf{16})_i(\mathbf{16})_j(\mathbf{120}) + y_{ij}^{126}(\mathbf{16})_i(\mathbf{16})_j(\overline{\mathbf{126}}) \qquad (2.6)$$

due to the famous decomposition $\mathbf{16} \otimes \mathbf{16} = \mathbf{10}_s \oplus \mathbf{120}_s \oplus \overline{\mathbf{126}}_s$. As has been widely stated, the minimal Higgs content in order to generate Dirac masses for quarks and leptons and Majorana masses for right-handed neutrinos is to have two Higgs bosons in the $\mathbf{10}$ representation of SO(10) and a Higgs in the $\overline{\mathbf{126}}$ representation.

The Yukawa couplings y_{ij}^{10} associated with $\mathbf{10}$ representations give

$$(m^d)^T = m^e, \quad m^u = m_{LR}^\nu, \qquad (2.7)$$

while the $\overline{\mathbf{126}}$ gives mass only to right handed neutrinos.

Prior to the Super-Kamiokande experiments [18], there were successful models which could reproduce fermion masses with this Higgs content, however without explanation of the exact relations between the elements of the same Yukawa matrix. With the current data of the neutrino oscillation experiments [16], now it is clear that the minimal content must be extended to fit neutrino masses. This can be done by adding more $\mathbf{10}$, $\mathbf{120}$ or $\mathbf{126}$ Higgs fields or by adding non-renormalizable operators or other elements in the theory beyond a GUT.

If we consider non-renormalizable operators we can alter the structure of the Yukawa matrices. The Lagrangian of these operators is

$$\mathcal{L}_{M(ht)} = \mathcal{O}^{ij} y_{ij}^{\mathcal{O}}, \quad \mathcal{O}_{ij} \equiv \mathbf{16}_i \frac{R_1}{M_1} ... \frac{R_k}{M_k} \mathbf{10} \frac{R_{k+1}}{M_{k+1}} ... \frac{R_\ell}{M_\ell} \mathbf{16}_j, \qquad (2.8)$$

where R's are possible representations coupling to $\mathbf{10}$ and $\mathbf{16}$ and $y_{ij}^{\mathcal{O}}$ is their corresponding Yukawa matrix. An interesting case is the adjoint representation $\mathbf{45}$ whose vacuum expectation value (vev) can point to any direction in the space spanned by the $\mathbf{45}$ generators as long as it leaves the SM group $SU(3)_c \times SU(2)_L \times U(1)_Y$ unbroken. The space of $\mathbf{45}$ vev, that leaves unbroken the SM group, lies in the two dimensional subspace of $U(1)'$ generators of SO(10) that commute with $SU(3)_c \times SU(2)_L \times U(1)_Y$. For example when SO(10) is broken to the $SU(3)_c \times SU(2)_R \times SU(2)_L$ subgroup, the general form to which the $\mathbf{45}$ should be pointing is

$$\langle \mathbf{45} \rangle = (B - L + kT_{R3})M_{\mathbf{45}}. \qquad (2.9)$$

Successful and predictive scenarios can be obtained when incorporating flavour symmetries into GUTs. With a clever choice of a flavour symmetry, it may not be needed to invoke

additional Higgs fields beyond two **10**s and one $\overline{\textbf{126}}$ representations. However majority of such examples do require at least one non-renormalizable operator (see [19] for a review).

From the experimental information on V_{CKM} and the quark masses we can reconstruct the possible forms that the quark mass matrices, m^q, can acquire from the above considerations. However since we are just able to construct the left diagonalizations of m^q from V_{CKM} we need to make more assumptions on m^q. One of the most successful and well established assumptions is to consider that the mixings in each quark sector are small such that they remain small when combined in the V_{CKM}. This assumption can be naturally identified with a strong hierarchical structure of m^q.

The diagonalizing matrix of $Y^q = \sqrt{2}m^q/v_q$ can be parameterized as the multiplication of unitary matrices $U_{23}U_{13}U_{12}$ where each U_{ij} is a rotation matrix in the (i,j) sector including some phases. When Y^q is hierarchical, the angles corresponding to this parameterization can be in fact approximately identified with the angles obtained from the approximate multiplication [20], $U_{12R}^\dagger U_{13R}^\dagger U_{23R}^\dagger Y U_{23L}U_{13L}U_{12L} \approx Y_{\text{diag}}$. For the case of the up quarks, we have

$$y_u \approx |c_{12}^L c_{12}^R y_{11}'' - e^{i\phi_L}c_{12}^L s_{12}^R y_{12}'' - e^{i\phi_R}c_{12}^R s_{12}^L y_{21}'' + s_{12}^R s_{12}^L y_{22}''|$$
$$y_c \approx c_{12}^L c_{12}^R y_{22}'' + e^{i\phi_L}c_{12}^L s_{12}^R y_{12}'' + e^{i\phi_R}|c_{12}^R s_{12}^L y_{21}'' + s_{12}^R s_{12}^L y_{11}''|,$$
$$y_t \approx |y_{33}c_{23}^L c_{23}^R + c_{23}^R s_{23}^L y_{32}e^{i\phi_{23L}} + c_{23}^L s_{23}^R y_{23}e^{i\phi_{23R}} + s_{23}^L s_{23}^R y_{22}| \tag{2.10}$$

where y'' is the matrix after the unitary transformation in the $(2,3)$ and $(1,3)$ sectors. We are particularly interested in analyzing the behavior in the $(1,2)$ sectors. For small rotations we have $c_{12}^R = c_{21}^R \approx 1$, $s_{12}^R = y_{21}''/y_{22}''$ and $s_{12}^L = y_{12}''/y_{22}''$. When $y_{11}'' \ll y_{12}''y_{21}''/y_{22}''$, one gets

$$|y_u| \approx |s_{12}^L s_{12}^R y_{22}''| \quad |y_c| \approx |y_{22}''|, \tag{2.11}$$

which, for the symmetric case, gives the famous relation $s_{12}^u \approx \sqrt{m_u/m_c}$. This forms the basis of the Gatto-Sartori-Tonin relation [22]

$$V_{us} = \left|s_{12}^d - e^{i\Phi_1}s_{12}^u\right| \approx \left|\sqrt{\frac{m_d}{m_s}} - e^{i\Phi_1}\sqrt{\frac{m_u}{m_c}}\right|. \tag{2.12}$$

This relation is in good agreement (originally the contribution just from the down sector was assumed, but now the contribution from the up sector has become important) with the experimental value of V_{us}. In order to review the conditions for which $y_{11}'' \ll y_{12}''y_{21}''/y_{22}''$, let us give explicitly their expressions in terms of the small rotation angles and the original Yukawa couplings:

$$y_{11}'' \approx y_{11} - \frac{[s_{23}^R y_{12}e^{i\phi_{23}^R} + c_{23}^R y_{13}][s_{23}^L y_{21}e^{i\phi_{23}^L} + c_{23}^L y_{31}]}{y_{33}c_{23}^L c_{23}^R + O(y_{23}y_{32}/y_{33})}$$
$$\frac{y_{12}''y_{21}''}{y_{22}''} \approx \frac{[c_{23}^R y_{12} - s_{23}^R y_{13}e^{-i\phi_{23}^R}][c_{23}^L y_{21} - s_{23}^L y_{31}e^{-i\phi_{23}^L}]}{y_{22}c_{23}^L c_{23}^R - c_{23}^R s_{23}^L y_{32}e^{-i\phi_{23}^L} - c_{23}^L s_{23}^R y_{23}e^{-i\phi_{23}^R} + s_{23}^L s_{23}^R y_{33}}. \tag{2.13}$$

From these expressions it is clear that, if the original Yukawa matrix contains a zero in the $(1,1)$ position, then the inequality $y_{11}'' \ll y_{12}''y_{21}''/y_{22}''$ is immediately achieved because of

$y_{33} \gg y_{22}$ leading to the desired hierarchy between charm and top quark masses. However notice that y_{11} does not have to be exactly zero. As long as it is suppressed enough with respect to $y''_{12} y''_{21}/y''_{22}$ we can always have a relation of the type eq. 2.12. This condition is referred hereafter as the limit $m^u_{11} \to 0$. As we have seen it can be realized in a particular basis of the Yukawa couplings satisfying eq. 2.11, which is however a basis independent statement.

For a symmetric case, if we assume $y_{12} \ll y_{23}$ then in any of the cases of having $y_{22} \sim y_{23}$, $y_{22} \ll y_{23}$ or $y_{22} > y_{23}$, the second term in the first of the equations of eq. 2.13 will be smaller than the expression for y''_{22}. Thus the interesting case is when y_{11} is the leading contribution in y''_{11}. Let us take the case $y_{22} \sim y_{23}$ then the leading contribution in y''^2_{12}/y''_{22} will be simply y^2_{12}/y_{22} and hence we obey the condition $y''_{11} \ll y''_{12} y''_{21}/y''_{22}$, in terms of the original matrix elements: $y_{11} \ll y^2_{12}/y_{22}$.

For matrices of the form [20, 2, 23], where the elements Y^u_{12} and Y^u_{22} are respectively ϵ^3_u and ϵ^2_u, one simply needs to require $Y^u_{11} \ll \epsilon^4_u$. For matrices of the form [24], which are also symmetric and hence compatible with SO(10), one recovers the requirements of the element Y^u_{11} with the present analysis. In this texture, Y^u_{12} is $\epsilon^3_u \sim \lambda^6$, Y^u_{22} is zero but Y^u_{23} is ϵ^2_u, and hence $(y'')^2_{12}/y''_{22} = \epsilon^4$. Thus, by making $Y^u_{11} = O(\epsilon^4_u)$ in this case, an $O(1)$ correction to the relation $s^u_{12} \approx \sqrt{m_u/m_c}$ can be made.

2.2 Compatibility with the experimental information

Symmetric fits to the quark masses can be used as a guideline to construct models with underlying SO(10), or $SU(4)_c \times SU(2)_L \times SU(2)_R$, which after the SO(10) breaking assume a symmetric structure. However a small departure from symmetric matrices do not change the qualitative behavior of such fits and can be made compatible with SO(10) and flavour symmetries.

The fits of these matrices into the experimental information can be made in many ways depending on our theoretical assumptions. A minimal choice is to assume that the supersymmetric corrections to the quark masses will not have a strong impact on the *ratio* of masses that we use for the fit. We also assume that the Yukawa matrices are the only source of CP violation and that the contributions from the transformation of the squared soft mass matrices of the supersymmetric particles to the basis where the Yukawa matrices are real and diagonal are negligible. We specify this last requirement because we can choose to fit the quark mass matrices with ratios of masses and with the fits of the unitary triangle, where various CP violating experiments are taken into account.

With increasing precision in the determination of the fits of the unitary triangle in the SM, however, one has a very tight constraint on the parameters and one should not regard this as a final fit of a particular texture but as an indication of the current compatibility of such texture with the theoretical assumptions and experimental information. The purpose of the present analysis is to clarify the consequences of having a negligible Y^u_{11} entry, giving the relation $s^u_{12} \approx \sqrt{m_u/m_c}$, and furthermore a low range of right handed neutrino masses, which turn out to be incompatible with the standard thermal leptogenesis to produce the observed baryon asymmetry of the universe. For completeness we present in appendix (B)

the current fit of a symmetric texture with negligible Y_{11}^u element, compatible at 68% C.L. with up-to-date fits of the unitary triangle.

It is interesting to see that two different choices of symmetric matrices in the up sector, ([2–4, 6]) and ([24, 25]), give rise to different phenomenology and consequences, e.g., for leptogenesis. This is because we then have clearer selection criteria on how to single out the models, apart from direct fittings to experimental mass matrices and quark mixing. Further precision analyses in the unitary triangle will constrain more such possibilities.

For non-symmetric fits [21], we have a rather different situation because often m_{11}^q should be of the order of m_{12}^q to reproduce appropriate mixings and quark masses. The non symmetric fits can be used to construct models with underlying SU(5) symmetries (see for example [26]).

In the following, we will consider the limit $(m_{LR}^\nu)_{11} = m_{11}^u \to 0$ and explore its consequences for neutrino oscillations and leptogenesis.

2.3 Consequences of $(m_{LR}^\nu)_{11} \to 0$

2.3.1 General structure of the matrix of low energy neutrinos

The explicit form of the low energy neutrino mass components from eq. (2.1) is

$$
\frac{m_{ee}^\nu}{m_{\nu_3}} = \mathbf{r} c_{13}^2 s_{12}^2 + e^{2i(\delta-\sigma)} s_{13}^2 + \mathbf{t} c_{13}^2 c_{12}^2 e^{-2i\rho}
$$

$$
\frac{m_{e\mu}^\nu}{m_{\nu_3}} = s_{23} s_{13} c_{13} e^{i(\delta-2\sigma)} + \mathbf{r} c_{13} s_{12} \left(c_{12} c_{23} - \frac{s_{12} s_{13} s_{23}}{e^{i\delta}} \right) - \frac{\mathbf{t}}{e^{2i\rho}} c_{12} c_{13} \left(c_{23} s_{12} + \frac{c_{12} s_{13} s_{23}}{e^{i\delta}} \right)
$$

$$
\frac{m_{e\tau}^\nu}{m_{\nu_3}} = c_{23} c_{13} s_{13} e^{i(\delta-2\sigma)} - \mathbf{r} c_{13} s_{12} \left(\frac{c_{23} s_{12} s_{13}}{e^{i\delta}} + c_{12} s_{23} \right) + \frac{\mathbf{t}}{e^{2i\rho}} c_{12} c_{13} \left(s_{23} s_{12} - \frac{c_{12} s_{23} s_{13}}{e^{i\delta}} \right)
$$

$$
\frac{m_{\mu\mu}^\nu}{m_{\nu_3}} = s_{23}^2 c_{13}^2 e^{-2i\sigma} + \mathbf{r} \left(c_{12} c_{23} - \frac{s_{12} s_{13} s_{23}}{e^{i\delta}} \right)^2 - \frac{\mathbf{t}}{e^{2i\rho}} (c_{23} s_{12} + c_{12} s_{13} s_{23}/e^{i\delta})^2
$$

$$
\frac{m_{\mu\tau}^\nu}{m_{\nu_3}} = \frac{c_{23} s_{23} c_{13}^2}{e^{2i\sigma}} - \mathbf{r} \left(\frac{c_{23} s_{12} s_{13}}{e^{i\delta}} + c_{12} s_{23} \right) \left(c_{12} c_{23} - \frac{s_{12} s_{13} s_{23}}{e^{i\delta}} \right)
$$
$$
- \frac{\mathbf{t}}{e^{2i\rho}} \left(c_{23} s_{12} + \frac{c_{12} s_{23} s_{13}}{e^{i\delta}} \right) \left(s_{12} s_{23} - \frac{c_{12} c_{23} s_{13}}{e^{i\delta}} \right)
$$

$$
\frac{m_{\tau\tau}^\nu}{m_{\nu_3}} = \frac{c_{13}^2 c_{23}^2}{e^{2i\sigma}} + \mathbf{r} \left(\frac{c_{23} s_{12} s_{13}}{e^{i\delta}} + c_{12} s_{23} \right)^2 + \frac{\mathbf{t}}{e^{2i\rho}} \left(s_{12} s_{23} - \frac{c_{12} c_{23} s_{13}}{e^{i\delta}} \right)^2, \tag{2.14}
$$

where we have expressed the elements of m^ν in terms of the sub-indices e, μ and τ. In appendix (A) we have written the numerical central values of the angles, up to $\mathbf{t}$, s_{13}^2 and possible phase variations. From eq. A.1 we can see that in the limit of $s_{13} = 0$, all the numerical entries that are not multiplied by $\mathbf{t}$ in eq. 2.14 do not change its order of magnitude. On the other hand the complete form of m^ν in terms of a diagonal matrix $M_R = \text{diag}[M_1, M_2, M_3]$ and a general matrix m_{LR}^ν is given by

$$
m^\nu = \sum_i \frac{1}{M_i} \begin{bmatrix} m_{1i}^2 & m_{1i} m_{2i} & m_{1i} m_{3i} \\ m_{1i} m_{2i} & m_{2i}^2 & m_{2i} m_{3i} \\ m_{1i} m_{3i} & m_{2i} m_{3i} & m_{3i}^2 \end{bmatrix}. \tag{2.15}
$$

Now when $(m_{LR}^\nu)_{11} = m_{11} \to 0$, this matrix acquires a very simple form. Then we can simply identify the elements of eq. 2.14 with eq. 2.15 and find the form of M_i in terms of m_{ij} and the restrictions of its elements.

It is clear from eq. 2.14 that the contribution from $\mathbf{t}$ can become relevant just for m_{ee}^ν and $m_{e\tau}^\nu$. But since $\mathbf{t} < O(0.1)$ according to eq. 2.3, this contribution can be at most as the same order of the rest of the contributions in $(m_\nu)_{11}$ or $(m_\nu)_{13}$. From eq. 2.15 we can see that these elements are given by

$$m_{ee}^\nu = \frac{m_{12}^2}{M_2} + \frac{m_{13}^2}{M_3}, \quad m_{e\tau}^\nu = \frac{m_{12}m_{32}}{M_2} + \frac{m_{13}m_{33}}{M_3}. \tag{2.16}$$

Identifying certain elements in m_{LR}^ν, we can obtain the predicted ranges for the right-handed neutrino masses which will be derived in detail in the next subsection. In this subsection, we try to extract the general expressions for the parameters such as

$$M_1, \quad \frac{M_1}{M_2} \quad \text{or} \quad \frac{M_1}{M_3}, \quad \text{and} \quad \tilde{m}_1 = \frac{(Y^{\nu\dagger}Y^\nu)_{11}v^2}{M_1}, \tag{2.17}$$

which are relevant for leptogenesis. Note that m_{ee}^ν is the element which contains less parameters and so we can make less assumptions when deriving expressions for eq. 2.17. Then we have

$$M_2 = \frac{m_{12}^2[1 + \frac{M_2}{M_3}\frac{m_{13}^2}{m_{12}^2}]}{m_{\nu 3}[\mathbf{r}c_{13}^2 s_{12}^2 + e^{2i(\delta-\sigma)}s_{13}^2 + \mathbf{t}c_{13}^2 c_{12}^2 e^{-2i\rho}]}$$
$$\gtrsim 2 \times 10^{16} y_{12}^2 \frac{\sin^2\beta}{2} \left[1 + \frac{M_2}{M_3}\frac{m_{13}^2}{m_{12}^2}\right] \text{ GeV}, \tag{2.18}$$

where we have set the bound on M_2 by taking the numerical values of the first two contributions of the denominator in eq. 2.18.

From this relation we can study the behavior of the parameters in terms of

$$a \equiv \frac{M_2}{M_3}\frac{m_{13}^2}{m_{12}^2}. \tag{2.19}$$

For $a \ll 1$ we can see that the order of M_2 is determined just by y_{12}^2. In this case, considering the expressions of $m_{\mu\mu}^\nu + m_{\tau\tau}^\nu$, we obtain the ratio of M_1 to M_2 given by

$$\frac{M_1}{M_2} = \frac{m_{ee}^\nu(m_{21}^2 + m_{31}^2)}{(m_{\mu\mu}^\nu + m_{\tau\tau}^\nu)m_{12}^2(1 + a) - m_{ee}^\nu\left[a(m_{23}^2 + m_{33}^2)\frac{m_{12}^2}{m_{13}^2} + (m_{32}^2 + m_{22}^2)\right]}. \tag{2.20}$$

Then, we find that generic SO(10) models lead to $\tilde{m}_1$ given by

$$\tilde{m}_1 = \max[m_{\mu\mu}^\nu + m_{\tau\tau}^\nu, b\, m_{ee}^\nu], \quad \text{where}$$
$$b \equiv \frac{\left[a(m_{23}^2 + m_{33}^2)\frac{m_{12}^2}{m_{13}^2} + (m_{32}^2 + m_{22}^2)\right]}{m_{12}^2(1 + a)}. \tag{2.21}$$

We can consider the cases for $b \lesssim 1$ and $b \gg 1$. For $b \lesssim 1$, the order of magnitude of $\tilde{m}_1$ is fixed simply by $m_{\mu\mu} + m_{\tau\tau} \sim (10^{-2}, 10^{-1})\,\text{eV}$. For $b \gg 1$, $\tilde{m}_1 \sim b(10^{-3}, 10^{-2})\,\text{eV}$. This

points out an important consequence that the right-handed neutrino gets out of thermal equilibrium when it is very non-relativistic [see next section] because $m_* < \tilde{m}_1$ where

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_{\mathrm{P}}} \simeq 1.6 \times 10^{-3} \text{ eV} \qquad (2.22)$$

with $g_* = 225$ for the relativistic degrees of freedom in supersymmetric standard model. This brings a strong suppression to the CP asymmetry generated through the decay of the right-handed neutrino.

For $a \gg 1$, instead of the bound for M_2, we determine the bound for M_3, which can be obtained by taking the replacements: $M_2 \leftrightarrow M_3$ and $m_{13} \leftrightarrow m_{12}$ in eq. (2.18). Similarly, the corresponding $\tilde{m}_1$ is given by the relation (2.21) with exchanging the indices $2 \leftrightarrow 3$ for the expression of b.

This has the same behavior as eq. 2.21 except that in GUT models the natural assumption is to have $y_{33}^\nu = O(1)$ and hence the second factor bm_{ee}^ν is likely to be the dominant. Then this contribution to $\tilde{m}_1$ goes like $\tilde{m}_1 \sim m_{ee} y_{33}^{\nu 2}/y_{13}^{\nu 2}$ which could be significantly bigger than $m_* \sim 10^{-3}\,\text{eV}$.

In all the models [2−4] and [6] in the basis where M_R is diagonal[1] the condition eq. 2.11 is satisfied for the case of the Dirac coupling of neutrinos and then we can identify its behavior in terms of a and b. In [3], a particular realization of the case with $a \gg 1$ and $y_{3,3}^\nu = 1$ was explored and one gets $M_1 \sim 10^7\,\text{GeV}$ and the wash-out factor, $\sim 10^{-5}$, to the leptonic CP asymmetry in the decay of right-handed neutrinos. The other parameters can bee seen in table 1. In [2], one gets $a = 0$ because effectively only two right-handed neutrinos are taken into account and the mass of the lightest right handed neutrino is $M_1 = O(10^7)\,\text{GeV}$. In [4], $a \ll 1$ and $b \in (O(0.1), 1)$ and the mass of the lightest neutrino is $M_1 = O(10^{(7,8)})\,\text{GeV}$. A realization of the case with $a \ll 1$ and $b \sim 1$ was explored in [7] and [6] where it is also not possible to achieve a successful thermal leptogenesis, due to the washout factor, although the mass of the lightest right handed neutrino is of order $M_1 = O(10^{10})\,\text{GeV}$. In table 1 we have summarized the properties relevant for leptogenesis of these models and we will make more comments about them in section 2.3.3.

In the next section we take the limit

$$\text{s}_{13} \to 0, \quad \text{t} \to 0 \quad \text{and} \quad 1/\text{M}_3 \to 0, \qquad (2.23)$$

which gives $a \ll 1$ and $b \ll 1$ and allows to understand more deeply the connection of $(m_{LR}^\nu)_{11} \to 0$ with the low mass of M_1 in GUT models.

2.3.2 Limit of $\text{s}_{13} \to 0$, $t \to 0$ and contributions proportional to $1/M_3$ negligible

The goal of this analysis is to identify the possible shapes of m_{LR}^ν and M_R in the basis where M_R is diagonal and the form of the mixing angles in terms of their entries. We will

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[1]For the models [4, 3], this transformation has been performed since for those models M_R is not diagonal in the basis where the underlying symmetry is broken.

Model	Ref.	$M_R[\text{GeV}]$	$a \equiv \frac{M_2}{M_3}\frac{m_{13}^2}{m_{12}^2}$	b	$\tilde{m}_1$
BKOT	[4]	$\begin{bmatrix} 10^{(7,8)} & & \\ & 10^9 & \\ & & 10^{(14,15)} \end{bmatrix}$	II. $\sim 10^{-3}$ III. $\sim \times 10^{-4}$ IV. $\sim \times 10^{-4}$	3×10^2	$b\,m_{ee}^\nu \sim$ $(10^{-1}, 1)$ eV
CM	[3]	$\begin{bmatrix} 10^7 & & \\ & 10^9 & \\ & & 10^{12} \end{bmatrix}$	10	10^3	$b\,m_{ee}^\nu \sim$ $(1, 10)$ eV
DHR	[6]	$\begin{bmatrix} 10^{10} & & \\ & 10^{12} & \\ & & 10^{13} \end{bmatrix}$	10^{-5}	$O(1)$	$m_{\mu\mu}^\nu + m_{\tau\tau}^\nu \sim$ $m_{\nu_3} \lesssim 0.625$ eV
RV	[2]	$\begin{bmatrix} 10^7 & & \\ & 10^8 & \\ & & > 10^{10} \end{bmatrix}$	0	$O(1)$	$m_{\mu\mu}^\nu + m_{\tau\tau}^\nu$

Table 1: Models based on SO(10) which do not generate the observed amount of baryon asymmetry through the decay of the right handed neutrinos in thermal leptogenesis.

assume that the mixing of charged leptons is small and hence can be ignored. In the limit under consideration, eq. (2.14) becomes much simpler as follows:

$$m_\nu = m_{\nu_3} \begin{bmatrix} s_{12}^2 \mathbf{r} & c_{12}c_{23}s_{12}\mathbf{r} & -c_{12}s_{12}s_{23}\mathbf{r} \\ c_{12}c_{23}s_{12}\mathbf{r} & c_{12}^2 c_{23}^2 \mathbf{r} + s_{23}^2 e^{-2i\sigma} & -c_{12}^2 c_{23}s_{23}\mathbf{r} + c_{23}s_{23}e^{-2i\sigma} \\ -c_{12}s_{12}s_{23}\mathbf{r} & -c_{12}^2 c_{23}s_{23}\mathbf{r} + c_{23}s_{23}e^{-2i\sigma} & c_{12}^2 s_{23}^2 \mathbf{r} + e^{-2i\sigma}c_{23}^2 \end{bmatrix}. \quad (2.24)$$

When comparing the elements m_{ij}^ν of eq. 2.24 with those of eq. 2.15, we have six equations to solve, which are more than the low-energy parameters to determine: one mass ratio, two angles and one phase. Thus the elements of m^D and M_R are more restricted. By comparing the m_{ee}^ν component in eq. 2.24 and eq. 2.15, we can readily identify M_2:

$$M_2 = \frac{m_{12}^2}{m_{\nu_3}\mathbf{r}s_{12}^2}. \quad (2.25)$$

Considering the ratio $m_{e\tau}^\nu/m_{e\mu}^\nu$, we obtain an important relation:

$$t_{23} = -\frac{m_{32}}{m_{22}}. \quad (2.26)$$

Analogously the ratio $m_{\mu\tau}^\nu/m_{\tau\tau}^\nu$ leads to

$$t_{23} = \left[\frac{m_{21}m_{31} + \frac{M_1}{M_2}m_{22}m_{32}}{m_{31}^2 + \frac{M_1}{M_2}m_{32}^2}\right]\left[\frac{1 + c_{12}^2 t_{23}^2 \mathbf{r}e^{2i\sigma}}{1 - c_{12}^2 \mathbf{r}e^{2i\sigma}}\right]. \quad (2.27)$$

When we determine the ratio M_1/M_2 we can put a restriction on the elements m_{21} and m_{31} from eq. 2.26 and eq. 2.27. Now the solar mixing angle is given by the following simple equation;

$$t_{12} = \pm\frac{m_{12}}{m_{22}}\frac{m_{32}}{\sqrt{m_{22}^2 + m_{32}^2}} \quad (2.28)$$

which is obtained by considering $m_{ee}^\nu/m_{e\mu}^\nu$ and the relation $t_{23}^2 = m_{32}^2/m_{22}^2$. Adding $m_{\mu\mu}^\nu$ and $m_{\tau\tau}^\nu$ we obtain

$$\frac{M_1}{M_2} = \left[\frac{\mathbf{r}s_{12}^2}{m_{12}^2}\right]\left[\frac{m_{21}^2 + m_{31}^2}{\mathrm{e}^{-2i\sigma} + \mathbf{r}c_{12}^2(1 - t_{23}^2)}\right] \approx \left[\frac{\mathbf{r}s_{12}^2}{m_{12}^2}\right](m_{21}^2 + m_{31}^2)e^{2i\sigma}, \qquad (2.29)$$

where the last equality follows from eq. 2.28. Analogously by adding $m_{\mu\tau}^\nu$ and $m_{\tau\tau}^\nu$ we obtain

$$\frac{M_1}{M_2} = \left[\frac{\left[\frac{\mathbf{r}s_{12}^2}{m_{12}^2}\right][m_{21}m_{31} + m_{31}^2]}{\mathrm{e}^{-2i\sigma}c_{23}(c_{23} + s_{23}) + \mathbf{r}c_{12}^2\left(s_{23}(s_{23} - c_{23}) - \frac{t_{12}^2}{m_{12}^2}[m_{32}^2 + m_{22}m_{32}]\right)}\right]. \qquad (2.30)$$

Now dividing eq. 2.29 by eq. 2.30 we can find solutions for $p \equiv m_{21}/m_{31}$ and then all the parameters can be expressed in terms of two unknowns; m_{31} and m_{22}. It is illustrative to consider the limiting case of $t_{23} = 1$ given the fact that the atmospheric neutrino mixing is the best measured quantity; $t_{23}^2 = 1_{+0.3}^{-0.3}$. Note in this case that we have the two solutions $p = 0$ or $p = 1$ along with the following relations;

$$m_{32} = -m_{22} \quad \text{and} \quad m_{12}^2 = 2t_{12}^2 m_{22}^2. \qquad (2.31)$$

Equating eq. 2.26 and eq. 2.27 we find

$$\frac{M_1}{M_2} = -\frac{m_{31}^2}{m_{22}^2}k \quad \text{with} \quad k \equiv \frac{1}{2}[(1 - p) - (1 + p)c_{12}^2\mathbf{r}e^{2i\sigma}], \qquad (2.32)$$

which leads us from eq. 2.25 to

$$M_1 = \frac{-2m_{31}^2}{c_{12}^2\mathbf{r}m_{\nu_3}}k. \qquad (2.33)$$

Now we find that eq. 2.32 is indeed compatible with eq. 2.29 for $p = 1$.

Form of the matrix m_{LR}^ν. Allowing a deviation from the limiting case of $t_{23} = 1$, we can get more general values for the parameters. In any case, we note that given m_{22} we can determine the elements m_{32} and m_{12} through low energy observables; t_{12} and t_{23} as in eq. 2.28, and hence fix the scale of M_2 by using m_{ν_3} and $\mathbf{r}$. Although we cannot fix the values of m_{21} and m_{31} independently, we can fix their ratio p and satisfy all experimental constraints. Then we can see how the hierarchy of M_1 and M_2 depends on the ratio of m_{22}^2 and m_{31}^2.

That is, we determine the following structure of m_{LR}^ν

$$m_{LR}^\nu = \begin{bmatrix} 0 & \frac{\mathrm{t}_{12}}{\mathrm{c}_{23}}m_{22} & x_1 \\ pm_{31} & m_{22} & x_2 \\ m_{31} & -t_{23}m_{22} & x_3 \end{bmatrix}, \qquad (2.34)$$

where the elements of the third column cannot be determined due to the limit $1/M_3 \to 0$ we have taken, and we can see that all the entries in the second column are of comparable

order. As we mentioned before, the motivation for having $(m_{LR}^\nu)_{11} \to 0$ was linked more closely to the symmetric fits, which implies $m_{12} \sim m_{21}$. From eq. 2.29, we can write

$$\frac{M_1}{M_2} \approx \frac{m_{21}^2}{m_{12}^2} \mathbf{r} s_{12}^2 \frac{1+p^2}{p^2}, \qquad (2.35)$$

which leads us to a conclusion of $M_1/M_2 \sim 0.2$ with $p = 1$. In the following, we will give more precise determination of the right-handed neutrino masses further considering the SO(10) structure of m_{LR}^ν in eq. 2.34.

2.3.3 SO(10) and the scale of M_1

We started our discussion by considering an underlying SO(10) framework with the mass matrix relation (2.7). Then, it has been realized [2] that at least one non-renormalizable operator should be included in the Yukawa sector to get a successful fit to the quark and charged-lepton sectors. Generically the representations which break the SO(10) down to SU(5) or $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$ and then further down to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ do not give rise to the appropriate structures of fermion masses, and thus we need to add at least one non-renormalizable operator. As we mentioned, the non-renormalizable operator involving the **45** representation is quite useful since its vev can be aligned in the $B - L + kT_{R3}$ direction for $k \in \mathbf{Z}$ whose value is different for every species of fermions. As we discussed in section 2.1, fits to the quark masses [20]–[23], give a structure of the up-sector of the form

$$Y^u \sim \begin{bmatrix} u & v & w \\ v & x & y \\ w & y & z \end{bmatrix}, \quad u \ll v, \quad u < w \lesssim v, \quad v < x, \quad y \lesssim x \ll z \sim O(1), \qquad (2.36)$$

where the exact symmetry of the matrix is not necessary but the orders of magnitude in the elements Y_{ij}^u and Y_{ji}^u have to be the same. In the previous section 2.3.2, we determined the possible form of m_{LR}^ν which can be naturally understood in the context of the SO(10) models when $m_{31} \sim m_{22}$. There are two forms of Yukawa matrices for the Dirac neutrinos that have been exploited in the literature:

$$Y^\nu = \begin{bmatrix} u' & v'_{12} & w'_{13} \\ v'_{21} & x' & y'_{23} \\ w'_{31} & y'_{32} & z' \end{bmatrix} \to \begin{cases} u' \ll w'_{13} \sim w'_{31} \sim v'_{12} \sim v'_{21}, & v'_{12} \sim x' \sim y'_{32} \sim y'_{23} \ll 1 \\ u' \ll v'_{13} \sim v'_{31} \ll w'_{12} \sim w'_{21}, & w'_{12} \ll x' \sim y'_{32} \sim y'_{23} \ll 1 \end{cases}$$

$$(2.37)$$

Each of these has been justified within the context of SO(10) and a particular flavour symmetry. However we can see that only the first pattern of eq. 2.37 agrees with eq. 2.34. The models [4] (BKOT), [12] (CM) and [2] (RV) are examples of the first form of Y^ν in eq. 2.37. In [2], for instance, the behavior of $x' \sim w' \sim v'$, which is different from $v \ll y \lesssim x$ in the up-quark sector, was explained by the coupling of the elements $(2,2)$ and $(2,3)$ to a **45** representation whose vev is proportional to $B-L+2T_{R,3}$. This vev is different from zero for up-quarks while it vanishes for neutrinos and thus one is forced to take into account the

next leading contribution which has to be of the same order for the entries $(1,2)$ and $(1,3)$ leading to a large mixing solution. The model of [6] (DHR) is an example of the second case. It has also a coupling of a **45** representation in the $(2,3)$ sectors. But the difference with respect to the up-quark Yukawa couplings is given by the vev breaking the flavour symmetry $D_3 \times U(1) \times Z_2 \times Z_3$ and together with the other orders of magnitude in Y^ν a large mixing is achieved. In this model it is also important to note that the contributions proportional to $1/M_3$ to the oscillation neutrino masses become relevant. Due to this, the complete analysis of section 2.3.2 does not apply here, since we have derived eq. 2.34 assuming the contribution proportional to $1/M_3$ is negligible. Now, eq. 2.12 fixes the order of ϵ_u as follows;

$$\epsilon_u \equiv \frac{v}{x} \approx \sqrt{\frac{m_u}{m_c}} \sim (3,6) \times 10^{-2}, \quad \frac{x}{z} \approx \frac{m_c}{m_t} \approx \epsilon_u^2, \quad z = 1 \Rightarrow v \approx \epsilon_u^3. \tag{2.38}$$

For the first case of eq. 2.37, v' is straightforwardly related to v:

$$v' = v \sim \epsilon_u^3. \tag{2.39}$$

On the other hand, for the second case of eq. 2.37, v' is also related to the up-sector but due to the structure of $D_3 \times U(1) \times Z_2 \times Z_3$ it is given by

$$v' \sim \epsilon_u^2. \tag{2.40}$$

Once the scale of m_{12}^2 is fixed, we can determine the order of magnitude of M_2 with $y_{12}^\nu = c(5 \times 10^{-2})^3$,

$$M_2 \approx \frac{y_{12}^2 v^2 \sin^2 \beta / 2}{m_{ee}^\nu} = c^2 \sin^2 \beta \,(0.5, 2.4) \times 10^8 \text{GeV}, \tag{2.41}$$

where c is an $O(1)$ number and m_{ee}^ν is taken from eq. A.1 considering the normal hierarchical spectrum of low energy neutrinos, $\mathbf{t} \ll \mathbf{r}$, and the uncertainties in eq. 2.3 are taken into account. When $\mathbf{r} \sim \mathbf{t}$, non-trivial phases can make m_{ee}^ν very small and the above estimation of M_2 has to be considered just as a lower bound. For eq. 2.41 we have $a \ll 1$ and $b \ll 1$, and hence this estimation can be applied to the BKOT, CM and RM models. For the model DHR [6] having $y_{12} = 6.27 \times 10^{-3} = O(\epsilon_u^2)$, one gets much larger value: $M_2 \approx (0.3, 1.1) \times 10^{12} \text{GeV}$ which agrees with the value quoted in table 1. We have to stress that in this case the contributions proportional to $1/M_3$ are not negligible.

For the explicit case with $p = 1$ analyzed in section 2.3.2, the ratio of M_1/M_2 becomes

$$\frac{M_1}{M_2} = \frac{m_{31}^2}{m_{22}^2}(0.089, 0.19). \tag{2.42}$$

Thus, we get the ranges of $M_1 \approx (0.045, 0.46) \times 10^8$ GeV taking $\sin^2 \beta = 1$.

3. Baryogenesis through leptogenesis

3.1 Thermal leptogenesis

The CP violating asymmetry generated in the decay of heavy right handed neutrino into a Higgs boson and a left-handed lepton and its CP conjugated channel, and its supersym-

metric counterpart are

$$\epsilon_{N_i} = \frac{\Gamma_{N_i,l} - \Gamma_{N_i,\bar{l}}}{\Gamma_{N_i,l} + \Gamma_{N_i,\bar{l}}}, \quad \epsilon_{\tilde{N}_i} = \frac{\Gamma_{\tilde{N}_i,l} - \Gamma_{\tilde{N}_i,\bar{l}}}{\Gamma_{\tilde{N}_i,l} + \Gamma_{\tilde{N}_i,\bar{l}}} \tag{3.1}$$

where $\Gamma_{N_i,l} \equiv \Sigma_{\alpha,\beta}\Gamma(N_i \rightarrow l^\alpha H_d^\beta)$ and $\Gamma_{N_i,\bar{l}} \equiv \Sigma_{\alpha,\beta}\Gamma(N_{R_i} \rightarrow \bar{l}^\alpha \bar{H}_d^\beta)$ are the N_i decay rates into l and $\bar{l}$ respectively. For the right-handed sneutrino $\tilde{N}_i$ decay rates, one has the final states with the lepton l and slepton $\tilde{l}$. The $B - L$ asymmetry generated by the right handed (s)neutrino decays is given by

$$Y_{B-L} = -\eta_i[\epsilon_{N_i}Y_{N_i}^{\mathrm{eq}} + \epsilon_{\tilde{N}_i}Y_{\tilde{N}_i}^{\mathrm{eq}}] = \frac{79}{28}Y_B \tag{3.2}$$

where Y_B is the resulting baryon asymmetry converted from the Y_{B-L} by the electroweak sphaleron processes, and η_i is the efficiency factor that measures the number density of $N_i/\tilde{N}_i$ decays at low temperature $T \ll M_i$ and $Y^{\mathrm{eq}} = Y^{\mathrm{eq}}(T \gg M_i) = 135\zeta(3)/(4\pi^2 g_\star)$. For $g_\star = 225$ in the supersymmetric model, one gets $Y_{N_i}^{\mathrm{eq}} = 1.9 \times 10^{-3}$. Since N_i and $\tilde{N}_i$ give the same contributions in the supersymmetric limit; $\Gamma_{N_i} = \Gamma_{\tilde{N}_i} \equiv \Gamma_i$ and $\epsilon_{N_i} = \epsilon_{\tilde{N}_i} \equiv \epsilon_i$, one can express the final baryon asymmetry as

$$Y_B = 1.3 \times 10^{-3} \, \eta_i \, \epsilon_i \tag{3.3}$$

where the observation requires $Y_B \approx 10^{-10}$. We recall that the decay rate for the right-handed (s)neutrino is

$$\Gamma_i = \Gamma_{N_i,l} + \Gamma_{N_i,\bar{l}} = \frac{(Y_\nu^{D\dagger}Y_\nu^D)_{ii}M_{N_i}}{4\pi}. \tag{3.4}$$

Then the CP asymmetry can be expressed by

$$\epsilon_i = \frac{1}{8\pi}\sum_{j\neq 1}\frac{\mathrm{Im}\left[(Y_\nu^{D\dagger}Y_\nu^D)_{ji}^2\right]}{[Y_\nu^{D\dagger}Y_\nu^D]_{ii}}f\left(\frac{M_j^2}{M_i^2}\right), \tag{3.5}$$

where, whenever the hierarchy $M_{N_j}^2/M_{N_i}^2 = x$ is good enough, the function $f(x)$ is just $f(x) \sim -\frac{3}{2\sqrt{x}}$. In the above expression, we neglected the lepton flavor dependence in the CP asymmetry which will be taken into account in the next subsection.

In our case, the right-handed neutrinos are charged under the SO(10) gauge group and thus are in thermal equilibrium at high temperature. The efficiency factor η_i can be calculated given the value of $K_i \equiv \Gamma_i/H(T = M_i)$;

$$K_i = \frac{\tilde{m}_i}{m_*} = \frac{\tilde{m}_i}{1.6 \times 10^{-3} \text{ eV}} \tag{3.6}$$

where the effective neutrino mass $\tilde{m}_i$ is defined by $\tilde{m}_i = 4\pi\Gamma_i v^2/M_i^2$. If $K_i \lesssim 1$, the efficiency reached its maximum $\eta_i = 1$. However, when $K_i \gg 1$ as is the case in most SO(10) models, the inverse decay remains effective for $T < M_i$ and its decoupling temperature $z_i \equiv M_i/T_i$ is given by [28]

$$\frac{K_i}{4}z_i^3 e^{-z_i}\sqrt{1 + \frac{\pi}{2}z_i} \simeq z_i - 1 \tag{3.7}$$

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and the corresponding efficiency factor can be well approximately by the simple form

$$\eta_i \simeq \frac{2}{z_i K_i} \left(1 - e^{-z_i K_i/2} \right). \tag{3.8}$$

It is important for our case, eq. 2.34 and $y_{12}^\nu \sim O(\epsilon_u^3)$, to notice that the lepton asymmetry along the electron direction generated by the second lightest right-handed neutrino N_2 is not washed out by the lightest right-handed neutrino N_1 as we have $y_{11} \to 0$. Therefore, let us consider the possibility of a successful leptogenesis either from N_1 or N_2 whose effective neutrino masses are

$$\tilde{m}_1 = \frac{v^2}{M_1} \sin \beta^2 |y_{31}|^2 [1 + p^2] \approx m_{\nu_3} \approx 0.05 \text{eV}\,,$$

$$\tilde{m}_2 = \frac{v^2}{M_2} \sin \beta^2 |y_{22}|^2 \frac{1 + \text{t}_{12}^2}{\text{c}_{23}^2} \approx m_{\nu_2} \approx 0.009 \text{eV}\,. \tag{3.9}$$

Thus we get

$$(K_i, z_i) = \begin{cases} (31.3, 7.51) & \text{for} \quad i = 1 \\ (6.26, 4.96) & \text{for} \quad i = 2 \end{cases} \tag{3.10}$$

leading to

$$\eta_i = \begin{cases} 8.5 \times 10^{-3} & \text{for} \quad i = 1 \\ 6.4 \times 10^{-2} & \text{for} \quad i = 2 \end{cases} \tag{3.11}$$

On the other hand, one can readily check that we have

$$O(\epsilon_{1,2}) \sim \frac{3}{16\pi} y_{ij}^2 \frac{M_1}{M_2} \lesssim 10^{-10} \tag{3.12}$$

putting the numerical values determined by the procedure of section 2.3. This is too small to produce the required amount of baryon asymmetry, 10^{-10}, as expected from a general discussion with hierarchical neutrino mass spectrum [5]. For the case of the DHR model [6], since the structure of Y^ν is different and the contribution from the three right handed neutrinos is important then it would be possible to achieve an acceptable value of the CP violation parameter relevant for leptogenesis [7]. However, the DHR model requires $M_1 \approx 10^{10}$ GeV which may cause the gravitino problem as will be discussed in the following.

3.2 Soft leptogenesis

It has been pointed out [15] that the B-term soft supersymmetry breaking of the right-handed sneutrino provides an additional source of lepton number and CP violation, where the relevant couplings are given by

$$-\mathcal{L}_{\text{soft}} = (m_{\tilde{N}}^2)_i^j \tilde{N}_R^{*i} \tilde{N}_{Rj} + (a^{ije} \tilde{l}_{Lj} \tilde{e}_{Ri}^* h_d + a^{ijN} \tilde{l}_{Lj} \tilde{N}_{Ri}^* h_u + \text{h.c}) + \left(\frac{1}{2} (b_N)_j^i \tilde{N}_R^{*i} \tilde{N}_{Rj}^* + \text{h.c.} \right). \tag{3.13}$$

The effects of $b_N \equiv BM_R$ terms are usually ignored because they are assumed to be highly suppressed by the difference in scales of the typical supersymmetric masses, 10^3 GeV, with

respect to the masses of the singlet neutrinos, $M_R \geq 10^7$ GeV. It turns out that there is a region in the parameter space of B and M_R compatible with models for which the masses of right-handed neutrinos are as low as $M_R \sim (10^7 - 10^8)$ GeV. The non-vanishing value of the generated lepton asymmetry is a pure thermal effect since at $T = 0$ the generated lepton asymmetry in leptons cancels the one in sleptons:

$$\epsilon_{\tilde{N}_i \to \tilde{l}H_d} = -\epsilon_{\tilde{N}_i \to l\tilde{H}} = \frac{4\Gamma_{\tilde{N}_i}B}{4B^2 + \Gamma_{\tilde{N}_i}^2}\frac{[\mathrm{Im}A]}{M_i}, \tag{3.14}$$

where $A = a_N/Y^\nu$. At finite temperature $T \neq 0$, the difference between the fermion and boson statistics yields non-vanishing lepton and CP asymmetry of the form $\epsilon_i(z) = \epsilon_{\tilde{N}_i \to \tilde{l}H_d}\delta_{\mathrm{BF}}(z)$ where $\delta_{\mathrm{BF}}(z)$ can be approximated for $z \gg 1$ by the analytic function of $\delta_{\mathrm{BF}}(z) \equiv 2\sqrt{2}K_1(\sqrt{2}z)/K_1(z)$ with $K_1(z)$ is the modified Bessel function of the first kind [29].

Thus, in the soft leptogenesis, one gets the reduced efficiency defined by

$$\tilde{\eta}_i \approx 2\sqrt{2}\frac{K_1(\sqrt{2}z_i)}{K_1(z_i)} \times \eta_i \tag{3.15}$$

where z_i is the decoupling temperature of the inverse decay calculated before. For our case with $z_i = 7.51$ and 4.96, we get $\delta_{\mathrm{BF}}(z_i) = 0.105$ and 0.3 and thus

$$\tilde{\eta}_i \approx \begin{cases} 8.9 \times 10^{-4} & \text{for} \quad i = 1 \\ 1.9 \times 10^{-2} & \text{for} \quad i = 2 \end{cases}. \tag{3.16}$$

Therefore, we require $\epsilon_{\tilde{N}_i \to \tilde{l}H_d} \approx 8.6 \times 10^{-5}$ and 4.0×10^{-6} correspondingly for a successful leptogenesis. When $\Gamma_i < B$, we get

$$\epsilon_{\tilde{N}_i \to \tilde{l}H_d} \approx \frac{\Gamma_i}{B}\frac{\mathrm{Im}[A]}{M_i} = \frac{\tilde{m}_i M_i}{4\pi v^2}\frac{\mathrm{Im}[A]}{B}. \tag{3.17}$$

Note here that we have calculated the largest CP asymmetries in N_i decaying to each flavor l. That is, only the e asymmetry is much smaller than the above expression for the N_1 decay due to the property of $Y_{11}^\nu \to 0$. Other flavor asymmetries are almost the same, which can be seen from eq. (2.34). Now one can find that the lightest right-handed sneutrino $\tilde{N}_1$ cannot produce enough lepton asymmetry due to a strong wash-out suppression. However, in the case of $\tilde{N}_2$ with $M_2 = 10^8$ GeV, the desired value of $\epsilon_{\tilde{N}_2 \to \tilde{l}H_d} \approx 4.0 \times 10^{-6}$ is found to be achieved for the hierarchical choice of soft parameters; $\mathrm{Im}[A] \approx 1.7$ TeV and $B = 1$ GeV. A crucial feature of our scheme is that the lepton asymmetry generated by the N_2 decay is not erased out by the interactions involving N_1 only in the electron direction due to the property of $Y_{11}^\nu \to 0$. Thus, we are relying on the flavor-dependent leptogenesis which has been discussed recently in various contexts [30].

Note that the leptogenesis scale $\sim 10^8$ GeV can be marginally allowed in view of abundant unstable gravitinos which decay late and upset the standard prediction of the big-bang neucleosynthesis [31]. Recent analyses showed that the upper bound on the reheat temperature is $T_R = 2 \times 10^6 - 3 \times 10^8$ GeV for the typical gravitino mass range of $m_{3/2} = 10^2 - 10^3$ GeV, assuming the hadronic branching ratio of the gravitino decay is 10^{-3}.

We also remark that the bound on the reheat temperature can be loosened if the gravitino is stable and forms dark matter. In this case, the next lightest supersymmetric particle needs to be a stau and the reheat temperature up to $T_R = 10^{10}$ GeV can be acceptable [32].

4. Supergravity description of a small B term

It has been pointed out that the smallness of the B term may arise if we have for example a dynamical mechanism that sets $B = 0$ at the leading order by arranging a specific form of the superpotential ref. [33]. However, we find it difficult to achieve such a mechanism without introducing a fine tuning of parameters in the general supergravity context. In this scenario, the other ingredient to produce a small B term of the order $m_{3/2}^2/M_{N_1}$ it may be through a term $\int d\theta^4 X^\dagger X N_1 N_1$ as in [34]–[35].

To illustrate difficulties in a dynamical set up of $B = 0$ in supergravity, let us consider the superpotential suggested in ref. [33]

$$W = \mu(\Phi_i)NN + Af(\Phi_i)X + W', \tag{4.1}$$

where Φ_i can be an observable field such as an multiplet of SO(10) (that is, **126** as N can be **16**) or any field in the hidden sector. The minimization condition of the scalar potential $V = e^K[K^{i\bar{j}}F_i F_{\bar{j}} - 3|W|^2]$ reads $V_l = 0$ where

$$V_l = e^K \left[K_l^{i\bar{j}}F_i F_{\bar{j}} + K^{i\bar{j}}\left[F_i(\bar{W}K_{\bar{j}l}) + F_{\bar{j}}(W_{il} + K_i W_l + K_{il}W) \right] - 3W_l\bar{W} \right] + K_l V \tag{4.2}$$

is the derivative of the potential V with respect to a field l. Here we have defined $F_l = W_l + K_l W$. The b term coming from the scalar potential is given by

$$b = e^K \left[K^{i\bar{j}}F_{\bar{j}}(\mu_i + K_i\mu) - 3\mu\bar{W} + \underline{2\mu\bar{W}} \right] \tag{4.3}$$

where the last term comes from $|F_N|^2 = |2\mu N + K_N W|^2$ with the minimal kinetic term for N; $K_N = \bar{N}$. On the other hand the minimization condition for X is

$$0 = K^{i\bar{j}}\left[F_i(\bar{W}K_{\bar{j}X}) + F_{\bar{j}}(W_{iX} + K_i W_X + K_{iX}W) \right] - 3W_X\bar{W}, \tag{4.4}$$

assuming $K_X^{i\bar{j}} = 0$ and $V = 0$ at the minimum. The indices i and j in the above equation contain Φ_i and X. Assuming there is no mixing term between them in K (that is, $K_{\Phi_i\bar{X}} = 0$ and $K_{\Phi_i X} = 0$), we separate the X index to write

$$0 = K^{i\bar{j}}\left[F_{\bar{j}}(W_{iX} + K_i W_X) \right] - 3W_X\bar{W}$$
$$+ K^{X\bar{X}}\left[F_X\bar{W}K_{\bar{X}X} + F_{\bar{X}}(W_{XX} + K_X W_X + K_{XX}W) \right]. \tag{4.5}$$

Thus we need to arrange the second line of eq. 4.5 and the W_{iX} contribution to sum up to $2\mu\bar{W}$ to cancel the above b term in eq. 4.3. We find that, with an specific form of a Kähler potential, one can achieve such a condition which however requires a fine tuning of the parameters involved and does not have a real theoretical justification.

The simplest way to arrange the condition of $B = 0$ is to rely on the no-scale super-gravity models as in ref. [36]. For this, let us take the hidden sector field ϕ with a Kälher potential

$$K = -3\log(\phi + \phi^*),$$

and its Yukawa coupling to matter fields

$$Y_{10,120}(\phi) = e^{-c\phi}, \qquad Y_{126} = const. \tag{4.6}$$

This can be a consequence of an U(1) symmetry under which ϕ transforms like $\phi \to \phi + i\alpha$, and then **16.16.10** and **16.16.120** are charged but **16.16.$\overline{126}$** is not. The A terms associated to the Yukawa couplings are given by [37]

$$AY = -m_{3/2}(\phi + \phi^*)\partial_\phi Y. \tag{4.7}$$

Therefore, we obtain $A_{10,120} \sim m_{3/2}$ and $B = A_{126} = 0$ at the GUT scale.

On the other hand the smallness of the B term needed for a successful soft leptogenesis could follow simply from a tuning among various supersymmetry breaking terms, which is technically natural if it is stable under sub-leading corrections. It is amusing to realize that the B term receives an important radiative correction due to gauge interactions of the right-handed (s)neutrinos. In the context of SO(10), the right-handed (s)neutrinos have a coupling to a heavy gauge boson X and the corresponding gaugino $\tilde{X}$ which also obtains a supersymmetry breaking mass $m_{1/2}$. Specifically, the gauge coupling $\tilde{N}$–N–$\tilde{X}$ leads to the one-loop correction to the B term of $\tilde{N}$ which is given by

$$BM \approx \frac{\alpha}{4\pi}m_{1/2}M \log\frac{M_X}{M} \tag{4.8}$$

where M_X is the mass scale of the heavy gauge boson X or the $B - L$ symmetry breaking, for instance. Now, putting $\alpha = 1/30$, $M = 10^8$ GeV and $M_X = 10^{10}$ GeV, we find $B \approx 10^{-2}m_{1/2}$ which gives us the required value of $B \approx 1$ GeV for $m_{1/2} = 100$ GeV.

5. Conclusions

The motivation of our work was to understand why in many GUT models which describe successfully the right values of fermion masses and mixings ([2–4, 6]), it is not possible to achieve the observed baryon asymmetry through the decay of heavy right-handed neutrinos. Apart from the strong hierarchy in the neutrino Yukawa couplings Y^ν, two factors are important: first linking m^u and m^ν_{LR} (in the simplest case they could be the same), which is a general SO(10) GUT relation, and then having the special feature of $Y^\nu_{11} \to 0$. Starting from these conditions we reconstructed the general structure of Y^ν and the mass scales of two right-handed neutrinos which are compatible with the neutrino data as well as the GUT relations enforced by a certain flavor symmetry in the decoupling limit of the heaviest right-handed neutrinos.

Our analysis shows that the neutrino couplings associated with two light right-handed neutrinos are determined by $Y^\nu_{i1,j2} \sim \epsilon_u^3 \sim 10^{-4}$ while the right-handed neutrino masses

are of order $(10^7 - 10^8)\,\text{GeV}$. Conventionally, such a parameter region is far away from a successful leptogenesis unless a certain fine-tuning is arranged between two light right-handed neutrinos to resonantly enhance the resulting lepton asymmetry. However, the soft leptogenesis arising from the CP phases of A and B supersymmetry breaking soft parameters can work consistently with our picture although our parameters are in the strong wash-out regime of the lepton asymmetry. The basic ingredients for this to occur are (i) $Y_{11}^\nu \to 0$ and (ii) a resonance condition of $B \sim \Gamma$. The first property protects the electron asymmetry which is generated by the second lightest right-handed sneutrino whose wash-out factor is favorably smaller. Interestingly the resonance condition requiring $B \sim 1\,\text{GeV}$ can be a consequence of the gauge one-loop correction involving the coupling of the right-handed sneutrino to the heavy GUT gaugino. For the vanishing condition of the tree-level B term, one may invoke no-scale supergravity, as it is difficult to achieve a dynamical realization of $B = 0$ by arranging a specific form of the superpotential and Kähler potential, which requires to introduce a fine tuning of parameters in the general supergravity context.

Acknowledgments

A. Numerical form of m_ν

The order of magnitudes in m_ν can be illustrated in the case $m_{\nu_3} \gg m_{\nu_2} > m_{\nu_1}$, for which we determine numerically the approximate values of m_ν up to the value of $\mathbf{t} < \mathbf{r}$, eq. 2.4, and the limit $s_{13}^2 \leq 0.041$:

$$
\begin{aligned}
\frac{(m_\nu)_{ee}}{m_{\nu_3}} &= 0.0542 + s_{13}^2 + 0.719\mathbf{t} \\
\frac{(m_\nu)_{e\mu}}{m_{\nu_3}} &= 0.102(0.600 - 0.375\,s_{13}) + \frac{s_{13}}{\sqrt{2}} - 0.848(0.375 + 0.600s_{13})\,\mathbf{t} \\
\frac{(m_\nu)_{e\tau}}{m_{\nu_3}} &= -0.102(0.600 + 0.375\,s_{13}) + \frac{s_{13}}{\sqrt{2}} + 0.848(0.375 - 0.600s_{13})\mathbf{t} \\
\frac{(m_\nu)_{\mu\mu}}{m_{\nu_3}} &= \frac{1}{2} + 0.193(0.600 - 0.376\,s_{13})^2 - (0.375 - 0.600\,s_{13})^2\mathbf{t} \\
\frac{(m_\nu)_{\mu\tau}}{m_{\nu_3}} &= \frac{1}{2} - 0.193(0.600 - 0.376\,s_{13})(0.600 - 0.376s_{13}) \\
&\quad -(0.375 + 0.600\,s_{13})(0.375 - 0.600\,s_{13})\mathbf{t} \\
\frac{(m_\nu)_{\tau\tau}}{m_{\nu_3}} &= \frac{1}{2} - 0.193(0.600 + 0.375s_{13})^2 + (0.375 - 0.600s_{13})^2\mathbf{t}.
\end{aligned}
\tag{A.1}
$$

B. Fit to a particular form of symmetric Yukawa matrices

B.1 Assumptions

A fit of textures for up and down Yukawa matrices of the form eq. 2.36 with Y_{11} negligible can be made compatible with the experimental information, under the following theoretical assumptions:

(a) Yukawa matrices are the only source of CP violation.

(b) We have a supersymmetric scenario which respects at low energy the constraints of unitarity of the CKM matrix.

(c) The supersymmetric corrections to the ratios $\frac{m_u}{m_d}$, $\frac{m_c}{m_s}$ and $\frac{m_s}{m_b}$ do not exceed the percentage of error on those ratios as quoted in table 2.

Since we have assumed (a) and (b), we must make the fits that test the unitarity of the CKM matrix in the Standard Model, where all experimental information has been taken into account, rather than to specific experiments. We use the classical fit [40], which takes into account the measurements of the following flavour violating processes:

$$\frac{|V_{ub}|}{|V_{cb}|}, \quad \frac{|V_{td}|}{|V_{ts}|}, \quad \epsilon_k, \quad \Delta m_{B_d}, \quad \Delta m_{B_s}, \quad \text{and} \quad \sin\beta. \tag{B.1}$$

These fits are a test of the unitarity of the CKM matrix in the Standard Model. That is, if all the experimental inputs in eq. B.1 are in agreement with the unitary of the CKM matrix, the statistical and systematic errors are under control on those measurements and there is no sensitivity to physics beyond the Standard Model in those processes then *after* the fit all these *fitted* quantities will agree with their input values at the 68% confidence level (C.L.). If some of them do not agree then there is an indication of either (i) the departure of the unitarity of the CKM matrix, (ii) a large correction from statistical or systematic errors in the experimental measurements, or (iii) a contribution from process beyond the standard model to the constraints eq. B.1 at the level of sensitivity at which the measurements and the analyses are performed. The other part of our experimental inputs comes from considering the following mass ratios

$$\frac{m_u}{m_d}, \quad \frac{m_c}{m_s}, \quad \frac{m_s}{m_b}, \tag{B.2}$$

at the scale M_Z, as well as the chiral perturbation parameter Q [39]:

$$Q = \frac{m_s/m_d}{\sqrt{1 - (m_s/m_d)^2}}. \tag{B.3}$$

We take into account the renormalization from the low scales, at which the quark masses are measured or computed according to experimental data, up to M_Z as

$$\eta_i \equiv \frac{m_i(M_Z)}{m_i(m_i)} \text{ for } i = c, \ b, \ t \ ; \qquad \eta_i \equiv \frac{m_i(M_Z)}{m_i(2 \text{ GeV})} \text{ for } i = u, \ d, \ s.$$

At two loops in QCD they can be estimated to be $\eta_c = 0.56$, $\eta_b = 0.69$, $\eta_t = 1.06$, $\eta_u = \eta_d = \eta_s = 0.65$.

The form of the Yukawa matrix eq. 2.36 is thought as being compatible with supersymmetric SO(10) models for which $\tan \beta$ is large, ($\sim$ 40-50). In this case the corrections to quark masses are not negligible, e.g., for the b quark can be up to 20% [38]. Thus strictly speaking we have

$$m_{f_d} = \sqrt{2} M_W \frac{y_{f_d}}{g} \cos \beta_S (1 + \varepsilon_{f_d} \tan \beta_S), \quad m_{f_u} = \sqrt{2} M_W \frac{y_{f_u}}{g} \sin \beta_S (1 + \varepsilon_{f_u} \tan \beta_S),$$

where the parameters $\varepsilon_{f,u}$ depend on the supersymmetric particles, such as charginos, neutralinos and gluinos, and y_{f_i} are eigenvalues of the Yukawa matrices. However we assume here that the ratios are not strongly affected by those corrections:

$$\frac{m_u}{m_c} = \frac{1 + \epsilon_u}{1 + \epsilon_c} \frac{y_u}{y_c} \equiv \frac{1}{r_{uc}} \frac{y_u}{y_c}, \quad r_{uc} \approx 1, \tag{B.4}$$

analogously for the other mass ratios considered.

Under the conditions of eq. 2.36 then we expect the angles of the left diagonalization matrices to be given as

$$
\begin{aligned}
s_{12}^u &= \sqrt{\frac{y_u}{y_c}} &\rightarrow& \quad \sqrt{\frac{m_u}{m_c} r_{uc}}, \\
s_{12}^d &= \sqrt{\frac{y_d}{y_s + y_d}} &\rightarrow& \quad \sqrt{\frac{\frac{m_d}{m_s} r_{KS}}{1 + \frac{m_d}{m_s} r_{ds}}}, \\
s_{23}^d &= \sqrt{\frac{\frac{Y_{22}^d}{Y_{33}^d} - \frac{y_s}{y_b}}{2\frac{y_s}{y_b} + 1}} &\rightarrow& \quad \sqrt{\frac{\frac{Y_{22}^d}{Y_{33}^d} - \frac{m_s}{m_b} r_{sb}}{2\frac{m_s}{m_b} r_{sb} + 1}}, ,
\end{aligned}
\tag{B.5}
$$

where r_{ab} are defined as in eq. B.4, and s_{12}^d now contains m_d in the denominator, which is a correction to the approximate formula $s_{12}^d = \sqrt{m_d/m_s}$. The angle s_{23}^d is obtained, of course, assuming that it is small and extracted from the relation,

$$s_{23}^d = \frac{Y_{23}^d}{Y_{33}^d} \quad \rightarrow \quad \frac{y_s}{y_b} = \frac{Y_{22}^d - Y_{33}^d s_{23}^{d2}}{Y_{33}^d (1 + 2 s_{23}^{d2})}.$$

The rest of the diagonalization angles are subject to the following conditions:

$$s_{13}^u \ll s_{23}^u \ll s_{12}^u, \qquad s_{13}^u \lesssim s_{13}^d \ll s_{23}^d, \qquad s_{23}^u \ll s_{23}^d. \tag{B.6}$$

If we assume that the phases of the elements Y_{23}^d and Y_{23}^u vanish or are the same, we can describe then the angles defining the CKM matrix in terms of two phases, ϕ_1, ϕ_2 [23]:

$$
\begin{aligned}
s_{12} e^{i\varphi_{12}} &= s_{12}^d - c_{12}^d s_{12}^u e^{i\phi_1} \\
s_{13} e^{-i\delta} &= s_{13}^d e^{i(\phi_2 - \varphi_{12})} - s_{12}^u s_{23}^d e^{i(\phi_1 - \varphi_{12})} \\
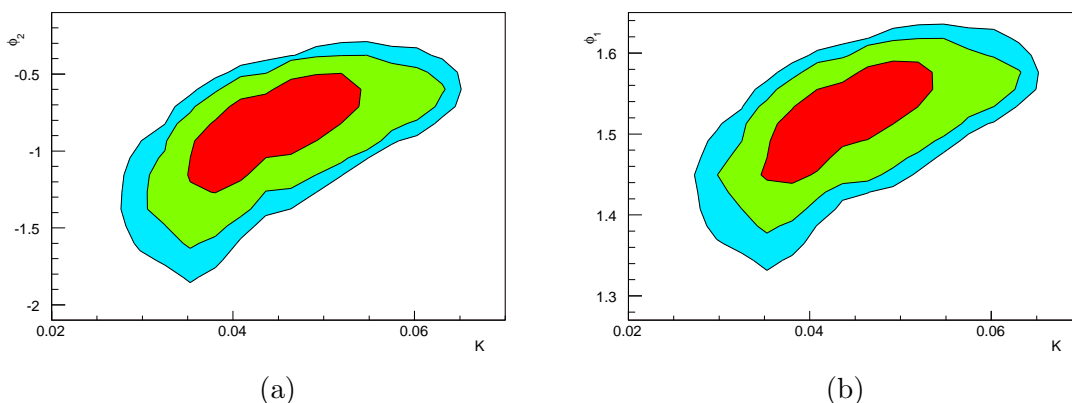s_{23} &= s_{23}^d,
\end{aligned}
\tag{B.7}
$$

Figure 1: 2D probabilities (a) for K and ϕ_1 and (b) for K and ϕ_2, the C.L. shown are at 68%, 95% and 99%.

where then ϕ_1 and ϕ_2 will be given as combination of the phases of the elements Y_{ij}^f, except for $(i,j) = (2,3)$ and for Y_{11}^u that we are neglecting. Hence the CKM elements are expressed in terms of

$$|V_{ub}| = |s_{12}^u s_{23}^d - s_{13}^d e^{i(\phi_2-\phi_1)}|,$$
$$|V_{cb}| = |s_{23}^d|,$$
$$\left|\frac{V_{td}}{V_{ts}}\right| = \left|\frac{s_{12}^d - c_{12}^d s_{12}^u e^{-i\phi_1}}{c_{12}^d} - \frac{1}{s_{23}^d}\left[s_{13}^d e^{-i\phi_2} - s_{12}^u s_{23}^d e^{-i\phi_1}\right]\right| = \left|t_{12}^d - \frac{s_{13}^d}{s_{23}^d}e^{-i\phi_2}\right|,,$$
$$|V_{us}| = |s_{12}^d - s_{12}^u c_{12}^d e^{i\phi_1}|,$$
$$\mathrm{Im}J = (s_{23}^d)^2 c_{12}^d \left[s_{12}^d s_{12}^u \sin\phi_1 + c_{12}^d s_{12}^u K \sin(\phi_1 - \phi_2) - s_{12}^d K \sin\phi_2\right], \qquad (B.8)$$

where $K \equiv s_{13}^d/s_{23}^d$. In eq. B.8, the quantities that are not given by the form of the matrix eq. 2.36 or by the conditions eq. B.6, are

$$s_{13}^d, \quad \phi_1 \quad \text{and} \quad \phi_2. \qquad (B.9)$$

Assuming $s_{13}^d \ll s_{23}^d$ and given eq. B.5, we have $s_{23}^d < s_{12}^d$. Also from the dependence of V_{td}/V_{ts} on s_{13}^d and s_{23}^d and from the fact that we are fitting to s_{23}^d directly because we are effectively making $V_{cb} = s_{23}^d$, we can fit to K and the phases ϕ_1 and ϕ_2. Here we choose to fit also ϕ_1 to check the level of compatibility of having $\phi_1 = \pi/2$.

B.2 Method of the fit

Note that we do not know all the entries in eq. B.8 and we have 4 CKM parameters and 3 mass ratios from which we can fit. Thus, instead of using a χ method, we use a Bayesian approach where we can obtain the combined probability distribution for K, ϕ_2 and ϕ_1, which is identified with the likelihood;

$$\mathcal{L}(K,\phi_1,\phi_2) \propto \int \prod_{j=1,M} f(\hat{c}_j | c_j(K,\phi_1,\phi_2,\{x_i\})) \times \prod_{i=1,N} f_i(x_i)\, dx_i \times f_0(K,\phi_1,\phi_2), \quad (B.10)$$

Input Values				
Constraints	Value $\pm$ Gaussian errors	Flat errors	% of error	Referen.
$|V_{ub}|$	$(3.683^{+0.106}_{-0.079}) \times 10^{-3}$			[41]
$|V_{cb}|$	$(41.61^{+0.62}_{-0.63}) \times 10^{-3}$			"
$|V_{td}|/|V_{ts}|$	$0.2003^{+0.0146}_{-0.0059}$			"
$|V_{us}|$	$0.22715^{+0.00101}_{-0.00100}$			"
$\text{Im}\{J\}$	$(2.91^{+0.25}_{-0.14}) \times 10^{-5}$			"
Varied Parameters.				
$\frac{m_u}{m_d}$	0.553 ± 0.043		7.7%	
$\frac{m_c}{m_s}$	11.3 ± 2.8		33.5%	
$\frac{m_s}{m_b}$	0.0213 ± 0.006		28%	
Q	22.7 ± 0.8			
$\frac{y^d_{22}}{y^d_{33}}$	0.036	± 0.014		

Table 2: Input values for constraints and varied parameters which are also fitted.

where $f(\hat{c}_j|c_j(K, \phi_1, \phi_2\{x_i\}))$ is the conditional probability density function (pdf) of the constraints $c_j = |V_{ub}|, |V_{cb}|, V_{us}, |V_{td}|/|V_{ts}|$ and $\text{Im}\{J\}$ given their dependence as functions of the texture parameters: s^d_{12}, s^u_{12}, c^d_{12}, s^d_{13} and s^d_{23} as well as the parameters and $x_i = \{m_u/m_d,\ m_c/m_s,\ m_s/m_b, Q\}$.

We have taken the values of the constraints $c_j = |V_{ub}|,\ |V_{cb}|, V_{us},\ |V_{td}|/|V_{ts}|$ and $\text{Im}\{J\}$ from the most recent CKM fitter results [41] , listed in table 2. The values of the observables

$$\sin 2\alpha, \quad \sin 2\beta, \quad \sin 2\gamma, \tag{B.11}$$

that we have to compare to our fit are then the values from the same CKM fitter results. Here

$$\alpha = \text{Arg}[-V_{31}V^*_{33}/(V_{11}V^*_{13})],$$
$$\beta = \text{Arg}[-V_{21}V^*_{23}/(V_{31}V^*_{33})]$$
$$\gamma = \text{Arg}[-V_{11}V^*_{13}/(V_{21}V^*_{23})],$$

are the angles of the unitary triangle of the CKM matrix V.

For our fit then the fitted values of the parameter of texture eq. 2.36 will be in agreement with the fits of the CKM fitter of the unitary triangle, assuming that the supersymmetric contributions are not relevant at the sensitivity at which the parameters eq. B.1 are related to their SM counterpart.

B.3 Results and comments

In figure 1, we show the results for the 2D probabilities of the parameter K versus ϕ_2 and ϕ_1. Given the method used for this fit we expect at least a 95% C.L. compatibility

Other values from the CKM fitter				
Parameter	CKM value $\pm 1\sigma$ C.L.	$\pm 2\sigma$ C.L.	Direct exp. value $\pm 1\sigma$ C.L.	$\pm 2\sigma$ C.L.
α	$99.0^{+4.0}_{-9.4}$	$^{+8.0}_{-17.9}$	$92.6.0^{+10.7}_{-9.3}$	$^{+27.1}_{-15.7}$
β	$22.03^{+0.72}_{-0.62}$	$^{+1.69}_{-1.27}$	$21.23^{+1.03}_{-0.99}$	$^{+2.09}_{-1.96}$
γ	$59.0^{+9.2}_{-3.7}$	$^{+18.0}_{-7.3}$	60.0^{+38}_{-24}	$^{+62}_{-39}$

Table 3: Relevant information from experiments and from the CKM fitter [41]. The later are included indirectly in the fit because they are not used as constraints.

Output Values	
Parameter	Value $\pm$ errors
K	$0.044^{+0.005}_{-0.007}$
ϕ_1	$1.530^{+0.050}_{-0.030}$
ϕ_2	$-0.877^{-0.162}_{+0.271}$
$\frac{y^d_{23}}{y^d_{33}}$	$0.023^{-0.065}_{+0.066}$
$|V_{ub}|$	$(3.75^{+0.096}_{-0.067}) \times 10^{-3}$
$|V_{cb}|$	$(42.25^{+0.58}_{-0.60}) \times 10^{-3}$
$|V_{td}|/|V_{ts}|$	$0.2046^{+0.0096}_{-0.0057}$
$|V_{us}|$	$0.22751^{+0.00091}_{-0.00081}$
$\text{Im}\{J\}$	$(3.02^{+0.18}_{-0.11}) \times 10^{-5}$
α	$(82.14^{+8.4}_{-8.5})^o$
β	$(21.83^{+2.3}_{-1.83})^o$
γ	$\approx \delta = (60.3^{+5.5}_{-5.3})^o$

Table 4: Our output values.

with what we have fitted. If that is not the case then obviously one of our theoretical assumptions should be modified. However we do have a 68% C.L. compatibility of all of our input values with the output (fitted) information. We show in table 2 the input values used for the fit. We have chosen to use the results of the CKM fitter collaboration [41] (which provides the SM fits to the Particle Data Group). In table 3 we show for comparison the different values of the unitary angles. In table 4 we show the results of our fit.

We can notice indeed that while the output angles β and γ are fitted in great agreement with the inputs, the angle α tends to be lower than the CKM fitter central value, although compatible at 68% C.L. This tendency could be due to the fact that the unitary triangle (UT) fit itself have shown consistently during the last 5 years a difficulty in fitting the unitary condition itself: $\pi = \alpha + \beta + \gamma$ (which is used in such fits with respect to the direct measurements) as we can immediately see also in table 3. Except for the fits of last year, all the angles of the UT fits have nevertheless being in agreement with that condition at 68% C.L. But there is still the possibility that there could be a sizable beyond the SM contribution that could show in future analyses, therefore changing the contribution of the

SM values to the fitted values of the unitary angles. Note also that the central value of the direct experimental value of α is lower than the central value of the CKM fitter and the errors are comparable.

Another thing to consider, of course, is that the model based in eqs. (2.36), (B.5) and (B.7) would need to include corrections or modifications. Among them are the supersymmetric corrections to the quark masses that should be carefully taken into account, or deviations from the symmetric textures. These deviations are indeed formally present since the symmetric structure of the mass matrices is valid just at the GUT scale and may get sizable modification by the RGE running to the electroweak scale. Given the increasing precision in the determination of the unitary triangle fits, this running should be taken into account. The running effects give a correction to the relation s_{12}^u of the form $s_{12}^u \approx 1/r\sqrt{m_u/m_c}$, where r is a parameter of $O(1)$ measuring the slight non-symmetry of the elements $|Y_{12}^u|$ and $|Y_{21}^u|$. Another possible modification is the one pointed out in [24], namely allowing the contribution of Y_{11}^u to become non negligible. This produces the same relation of $s_{12}^u \approx 1/r\sqrt{m_u/m_c}$ with r depending on the non negligible Y_{11}^u element. A separate question, independent of the fit itself, is whether this fit is compatible with a particular realization of a horizontal symmetry, like the one proposed in [23].

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